Power-law inflation with a nonminimally coupled scalar field

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We consider the dynamics of power-law inflation with a nonminimally coupled scalar field ϕ. It is well known that multiple scalar fields with exponential potentials $V(\phi) = V_0 \mathrm{exp}(-\sqrt{16\pi/pm_{\rm pl}^2}\phi)$ lead to an inflationary solution even if the each scalar field is not capable to sustain inflation. In this paper, we show that inflation can be assisted even in the one-field case by the effect of nonminimal coupling. When ξ is positive, since an effective potential which arises by a conformal transformation becomes flatter compared with the case of $\xi = 0$ for $\phi > 0$, we have an inflationary solution even when the universe evolves as non-inflationary in the minimally coupled case. For the negative ξ, the assisted inflation can take place when ϕ evolves in the region of $\phi < 0$.

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I. INTRODUCTION

The idea of inflation is one of the most reliable concepts in modern cosmology [1,2]. It can solve the horizon and flatness problem in the standard big bang cosmology, and also provides us the seeds of the large scale structure. The inflationary period proceeds while a scalar field called *inflaton* slowly evolves along a sufficiently flat potential.

It has been generally considered that only one scalar field determines the dynamics of inflation even if there are many scalar fields present in the inflationary epoch. However, Liddle, Mazumdar, and Schunck [3] recently showed that we have an inflationary solution with exponential potentials [4–6] in the multi-scalar field case even if individual fields do not possess flat potentials to lead to inflation. It was demonstrated that scaling solutions for exponential potentials with different slopes are the late-time attractors, and Malik and Wands confirmed this fact by choosing an appropriate rotation in field space [7]. Copeland, Mazumdar, and Nunes examined the generalized assisted inflation where the cross-coupling terms exist between scalar fields [8]. Exponential potentials often arise in the effective four-dimensional models induced by Kaluza-Klein theories. Kanti and Olive [9] investigated the assisted chaotic inflation with multiple scalar fields in higher-dimensional theories. The dynamics and density perturbations in assisted chaotic inflation was studied by Kaloper and Liddle [10]. Since most models in realistic higher-dimensional theories give rise to steep potentials by which inflation is hard to realize, the assisted mechanism by multiple scalar fields plays an important role for the realization of inflation.

From a viewpoint of quantum field theory in curved spacetime, it is natural to consider that the inflaton field ϕ couples nonminimally to the spacetime curvature R with a coupling of $\xi R\phi^2/2$. In the new inflation model [11], the existence of nonminimal coupling prevents inflation in some cases, because the flatness of the potential of inflaton is destroyed around $\phi = 0$. In the chaotic inflation model [12] with a nonminimally coupled inflaton field, Futamase an Maeda [13] investigated the constraint of the coupling ξ in two potentials of $V(\phi) = m^2\phi^2/2$ and $V(\phi) = \lambda\phi^4/4$. They found that ξ is restricted as $|\xi| \lesssim 10^{-3}$ to lead to sufficient inflation in the massive inflaton model. On the other hand, the constraint of ξ is absent in the self coupling model with negative ξ, and inflation is supported for larger values of $|\xi|$. Fakir and Unruh examined this self-coupling model with a strong negative nonminimal coupling $|\xi| \gg 1$ [14], and found that the fine tuning problem of the self-coupling λ is relaxed by such large values of $|\xi|$. Several

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authors studied scalar density perturbations [15–17] and tensor gravitational waves [18,19] during inflation in this model. In the context of preheating after inflation, we have showed that the fluctuation of inflaton can be enhanced nonperturbatively by nonminimal coupling [20].

In this paper, we consider power-law inflation with a nonminimally coupled inflaton field. What we are concerned with is whether power-law inflation is supported or not with the presence of nonminimal coupling. If the assisted mechanism works even in the one-field case, it is expected that this will also occur in the multi-field case. For a more complicated study of the multi-field case in the future, in this paper, we clarify in what values of ξ assisted inflation is realized by nonminimal coupling in the one-field case.

This paper is organized as follows. In the next section, we explain the model of power-law inflation with a nonminimally coupled inflaton field ϕ. In Sec. III, the dynamics of inflation is investigated in both cases of $\xi > 0$ and $\xi < 0$. We show in what cases the assisted inflation takes place by the effect of nonminimal coupling. We present our conclusions and discussions in the final section.

II. BASIC EQUATIONS

We study a model where an inflaton field ϕ is nonminimally coupled with a scalar curvature R:

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) - \frac{1}{2}\xi R\phi^2\right], \tag{2.1}$$

where $\kappa^2/8\pi \equiv G = m_{\rm pl}^{-2}$ is Newton's gravitational constant, and ξ is a coupling constant. In this paper, we consider an exponential potential $V(\phi)$ which is described by

$$V(\phi) = V_0\, \exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\phi}{m_{\rm pl}}\right), \tag{2.2}$$

where V_0 and p denote the energy scale which has the dimension of [mass]^4, and the power of inflation, respectively.

In the case of $\xi = 0$, we have an inflationary solution for $p > 1$,

$$a(t) \propto t^p, \tag{2.3}$$

where $a(t)$ is the scale factor. However, inflation does not take place for $p \le 1$ in the case of a single scalar field. Liddle et al. [3] showed that inflation proceeds for the case of multi-scalar fields even if the individual scalar field has the power less than unity. This cooperative behavior was termed *assisted inflation*. In this paper, we investigate whether nonminimal coupling will assist inflation or not in the single field case. The multi-scalar field case will be discussed elsewhere [21].

We find from the Lagrangian (2.1) that the effective gravitational constant $G_{\rm eff}$ depends on the value of the inflaton field,

$$G_{\rm eff} = \frac{G}{1-\phi^2/\phi_c^2}, \quad \text{with} \quad \phi_c^2 \equiv \frac{m_{\rm pl}^2}{8\pi\xi}. \tag{2.4}$$

In order to connect to our present universe, $G_{\rm eff}$ needs to be positive for the case of the positive ξ, which yields

$$|\phi| < \phi_c = \frac{m_{\rm pl}}{\sqrt{8\pi\xi}}. \tag{2.5}$$

When ξ is negative, such a constraint is absent.

We obtain the following field equations from the Lagrangian (2.1),

$$\frac{1-\xi\kappa^2\phi^2}{\kappa^2}G_{\mu\nu} = (1-2\xi)\nabla_\mu\phi\nabla_\nu\phi - \left(\frac{1}{2}-2\xi\right)g_{\mu\nu}(\nabla\phi)^2 - g_{\mu\nu}V(\phi) + 2\xi\phi(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\phi, \tag{2.6}$$

$$\Box\phi - \xi R\phi - V_{,\phi} = 0, \tag{2.7}$$

where $\Box$ and $V_{,\phi}$ are defined as $\Box \equiv \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)/\sqrt{-g}$, $V_{,\phi} \equiv \partial V/\partial\phi$ respectively. Although we can analyze the evolution of the system by Eqs. (2.6) and (2.7), it is rather complicated due to the existence of nonminimal coupling. It is convenient to transform to the Einstein frame by performing a conformal transformation

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \tag{2.8}$$

where $\Omega^2 \equiv 1-\xi\kappa^2\phi^2$. Then we obtain the following equivalent Lagrangian:

$$\mathcal{L} = \sqrt{-\hat{g}}\left[\frac{1}{2\kappa^2}\hat{R} - \frac{1}{2}F^2(\hat{\nabla}\phi)^2 - \hat{V}(\phi)\right], \tag{2.9}$$

where variables with a caret denote those in the Einstein frame, and

$$F^2 \equiv \frac{1-(1-6\xi)\xi\kappa^2\phi^2}{(1-\xi\kappa^2\phi^2)^2}, \tag{2.10}$$

$$\hat{V}(\phi) \equiv \frac{V(\phi)}{(1-\xi\kappa^2\phi^2)^2}. \tag{2.11}$$

Introducing a new scalar field Φ as

$$\Phi \equiv \int F(\phi)d\phi, \tag{2.12}$$

the Lagrangian in the new frame is reduced to the canonical form:

$$\mathcal{L} = \sqrt{-\hat{g}}\left[\frac{1}{2\kappa^2}\hat{R} - \frac{1}{2}(\hat{\nabla}\Phi)^2 - \hat{V}(\Phi)\right]. \tag{2.13}$$

In this paper, we adopt the flat Friedmann-Robertson-Walker line element as the background spacetime;

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2 = \Omega^{-2}(-d\hat{t}^2 + \hat{a}^2(\hat{t})d\mathbf{x}^2). \tag{2.14}$$

Note that $\hat{t}$ and $\hat{a}$ are related with those in the original frame as

$$\hat{t} = \int \Omega dt, \quad \hat{a} = \Omega a. \tag{2.15}$$

The evolutions of the scale factor and the Φ field in the Einstein frame yield

$$\hat{H}^2 \equiv \left(\frac{\hat{a}_{,\hat{t}}}{\hat{a}}\right)^2 = \frac{\kappa^2}{3}\left[\frac{1}{2}\Phi_{,\hat{t}}^2 + \hat{V}(\Phi)\right], \tag{2.16}$$

$$\Phi_{,\hat{t}\hat{t}} + 3\hat{H}\Phi_{,\hat{t}} + \hat{V}_{,\Phi} = 0, \tag{2.17}$$

where $,\hat{t} \equiv d/d\hat{t}$ and

$$\hat{V}_{,\Phi} \equiv \frac{d\hat{V}}{d\Phi} = \frac{4\xi\kappa^2\phi - \sqrt{16\pi/pm_{\rm pl}^2}(1-\xi\kappa^2\phi^2)}{(1-\xi\kappa^2\phi^2)^2\sqrt{1-(1-6\xi)\xi\kappa^2\phi^2}}V_0\,\exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\phi}{m_{\rm pl}}\right). \tag{2.18}$$

Note that $\phi_{,\hat{t}}$ and $\Phi_{,\hat{t}}$ are related by Eq. (2.12) as

$$\phi_{,\hat{t}} = \frac{1-\xi\kappa^2\phi^2}{\sqrt{1-(1-6\xi)\xi\kappa^2\phi^2}}\Phi_{,\hat{t}}. \tag{2.19}$$

We can know the behavior of the scale factor and the inflaton field by Eqs. (2.16) and (2.17) with an effective potential (2.11) in the Einstein frame. Transforming back to the original frame by making use of the relation (2.15), we can judge in what cases nonminimal coupling assists inflation. In what follows, we will investigate these issues in details.

III. ASSISTED INFLATION WITH A NONMINIMALLY COUPLED SCALAR FIELD

In this section, we study the dynamics of the system in both cases of the positive and negative ξ. Let us first review the case of $\xi = 0$ for comparison. In this case, making use of the slow-roll conditions $\dot{\phi}^2 \ll V$, $\ddot{\phi} \ll 3H\dot{\phi}$, Eqs. (2.16) and (2.17) are approximately written as

$$H^2 \approx \frac{\kappa^2}{3} V_0 \exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\phi}{m_{\rm pl}}\right), \tag{3.1}$$

$$3H\dot{\phi} \approx -\sqrt{\frac{16\pi}{p}}\frac{V_0}{m_{\rm pl}} \exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\phi}{m_{\rm pl}}\right), \tag{3.2}$$

where we dropped a caret in the Hubble parameter. Then we find that ϕ and H evolve as

$$\phi \approx \sqrt{\frac{p}{4\pi}} \log\left[\sqrt{\frac{\kappa^2 V_0}{3}}\frac{t}{p} + \alpha\right] m_{\rm pl}, \tag{3.3}$$

$$H \approx \left(\frac{t}{p} + \beta\right)^{-1}, \tag{3.4}$$

where α and β are some constants which depend on initial values of the ϕ field. We obtain the power-law solution (2.3) by integrating Eq. (3.4). When p is greater than unity, the potential (2.11) is flat enough to lead to inflation. Since the exponential potential does not have a local minimum, the inflationary phase continues forever. In realistic models of inflation, we need to consider some exit mechanisms from power-law inflation in order to lead to the successful reheating, radiation and matter dominant universe. In the following subsections, we consider the effect of nonminimal coupling in the dynamics of power-law inflation.

A. Case of $\xi > 0$

In this case, the inflaton field is required to lie in the region of (2.5). An important point with the existence of positive ξ is that an effective potential (2.11) in the Einstein frame has a local minimum at

$$\phi_* = \left(\sqrt{\frac{p}{4\pi} + \frac{1}{8\pi\xi}} - \sqrt{\frac{p}{4\pi}}\right) m_{\rm pl}. \tag{3.5}$$

Note that ϕ_* exists in the range of $0 < \phi_* < \phi_c$. The evolution of the scale factor depends on the initial condition of the inflaton field. When ϕ is close to the critical value $\pm\phi_c$, $\hat{V}$ is approximately written as [13]

$$\hat{V}(\Phi) \approx A \exp\left(2\sqrt{\frac{2}{3}}\kappa\Phi\right), \tag{3.6}$$

where A is a constant. This exponential potential has a power-law solution which is described by

$$\hat{a} \propto \hat{t}^{3/4}. \tag{3.7}$$

Going back to the original frame, the scale factor evolves as

$$a \propto t^{1/2}. \tag{3.8}$$

This means that we do not have an inflationary solution when ϕ is close to $\pm\phi_c$.

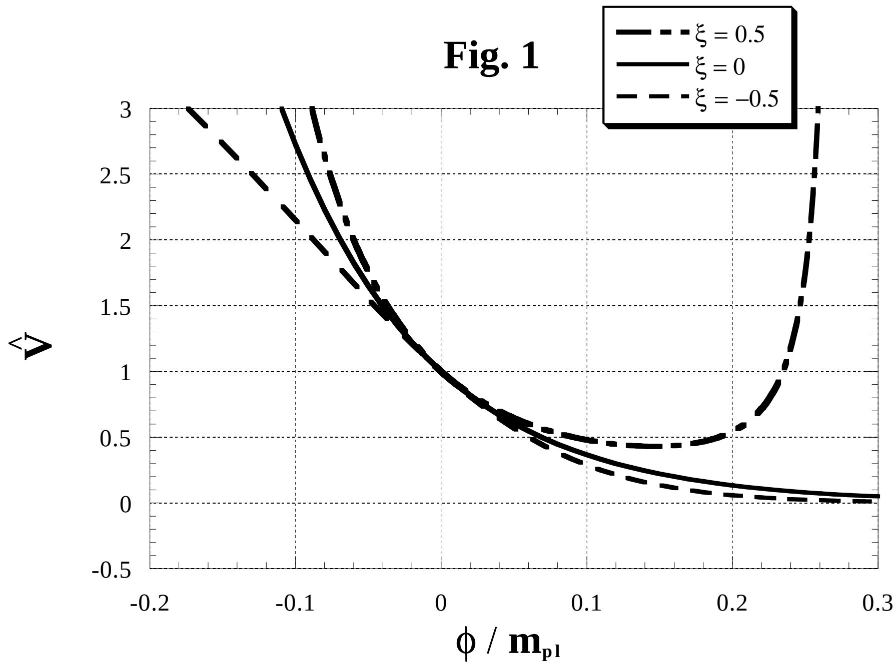


FIG. 1: An effective potential $\hat{V}(\phi)$ in the Einstein frame in the case of $\xi = 0.5, 0, -0.5$ with $p = 1/2$ (top, middle, bottom). When ξ is positive, since the potential $\hat{V}(\phi)$ is flat in the region of $\phi > 0$ compared with the $\xi = 0$ case, we can expect assisted inflation to occur in this region. When ξ is negative, assisted inflation can be realized in the region of $\phi < 0$.

Let us consider the case where the initial value of inflaton ($= \phi_i$) is in the range of $-\phi_c < \phi_i < \phi_*$. Since the slope of the effective potential $\hat{V}(\phi)$ becomes steeper than in the case of $\xi = 0$ for the values of $\phi < 0$ due to the existence of $1/(1 - \xi\kappa^2\phi^2)^2$ term (See Fig. 1), nonminimal coupling does not support inflation when ϕ rolls down in the region of $\phi < 0$. However, after inflaton passes through $\phi = 0$, we have a flatter effective potential compared with the case of $\xi = 0$. We can expect that the universe evolves as inflationary even for the value of $p \leq 1$. For example, consider the case of $p = 1/2$ and $\xi = 0.05$ with the initial value ϕ_i close to $-\phi_c = -0.892 m_{\rm pl}$. In Fig. 2, we show the evolution of the scale factor a as a function of t. The evolution of the inflaton field ϕ is also depicted in Fig. 3. Note that these variables are those in the original frame. We find that inflation does not take place in the region of $\phi < 0$. At the time of $\bar{t} \equiv m_{\rm pl} t \approx 0.1$, inflaton reaches $\phi = 0$. After that, the assisted mechanism due to nonminimal coupling begins to work, and the universe evolves as inflationary after $\bar{t} \approx 1.5$, which corresponds to the value of $\phi \approx 0.5 m_{\rm pl}$. Even in the case of $0 < \phi < 0.5 m_{\rm pl}$, the ξ effect makes the universe grow more rapidly than in the case of $\xi = 0$. Assisted inflation relevantly occurs in the region of $0.5 m_{\rm pl} \lesssim \phi \lesssim \phi_* = 0.714 m_{\rm pl}$. Inflaton slowly evolves in the flat region around $\phi = \phi_*$, and finally approaches the local minimum. In the case where inflaton is initially located in the region of $0 < \phi_i < \phi_*$, the assisted mechanism works from the beginning. After the field reaches the local minimum at $\phi = \phi_*$, it continues to stay there, and the inflationary phase does not terminate. Hence we need some exit mechanisms from inflation as in the minimally coupled case.

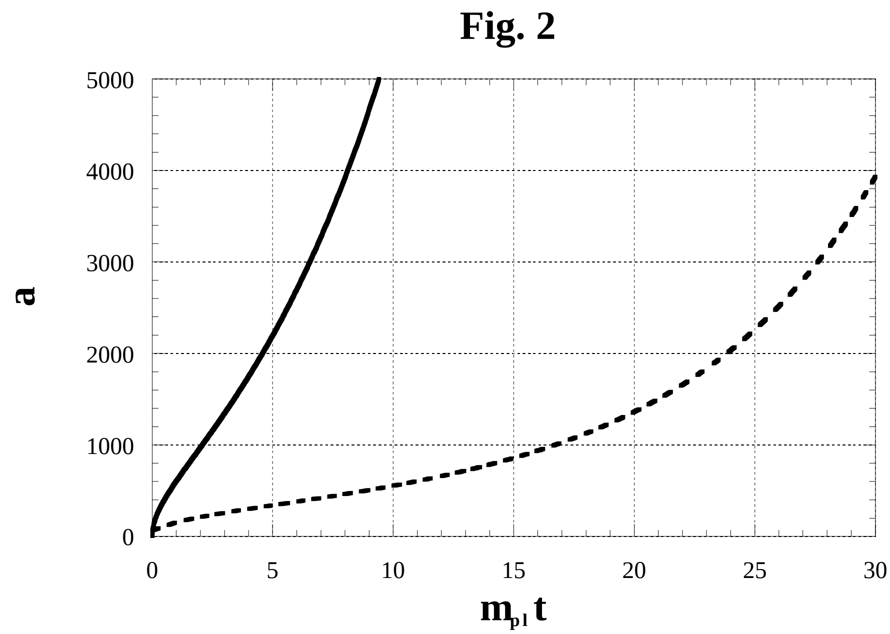


FIG. 2: The evolution of the scale factor a as a function of t in the case of $\xi = 0.05$ and $p = 1/2$ with the initial values of $\phi_i = -0.882 m_{\rm pl}$ (solid) and $\phi_i = 0.891 m_{\rm pl}$ (dotted). In both cases, we find that inflation begins to take place after the non-inflationary evolution at the initial stage.

Let us next examine the case where ϕ is initially in the range of $\phi_* < \phi_i < \phi_c$. If ϕ_i is close to the critical value ϕ_c, we have the non-inflationary solution (3.8) in any value of p and ξ. However, as ϕ approaches the value of ϕ_*, the assisted mechanism begins to work. Let us consider the case of $p = 1/2$ and $\xi = 0.05$ with the initial value of ϕ close to $\phi_c = 0.892 m_{\rm pl}$. We find in Fig. 2 the inflationary behavior after $\bar{t} \approx 5$, although the universe evolves deceleratedly as Eq. (3.8) at the initial stage. At $\bar{t} = 5$, the value of inflaton is $\phi = 0.880 m_{\rm pl}$ (See Fig. 3), which is close to the critical value ϕ_c. This means that we have the non-inflationary solution (3.8) only when ϕ is very close to ϕ_c. Nonminimal coupling drives inflation while inflaton slowly evolves in the flat region of $\phi_* < \phi \lesssim 0.880 m_{\rm pl}$. Finally, inflaton reaches the local minimum of the effective potential, as is the same with the case of $-\phi_c < \phi_i < \phi_*$.

Next, we investigate the case where p and ξ are changed. As is found by Eq. (3.5), ϕ_* decreases with the increase of ξ. Since the effective potential $\hat{V}(\phi)$ becomes flatter in the region of $0 < \phi < \phi_*$ as ξ increases, inflation is more easily realized in this region in spite of the decrease of ϕ_*. In the case of $\phi_* < \phi < \phi_c$, since the flat region around the local minimum becomes wider for larger value of ξ, assisted inflation occurs significantly except the case where ϕ is close to ϕ_c. This suggested that assisted mechanism works more efficiently with the increase of ξ in the region of $\phi > 0$, and we have numerically confirmed this fact.

If we choose the values of p with $p > 1$, inflation is always supported by the effect of positive ξ when inflaton evolves in the region of $\phi > 0$ as long as ϕ is not close to ϕ_c. On the other hand, since the slope of the effective potential becomes steeper with the decrease of p, we do not necessarily have inflationary solutions for the case of $p \leq 1$. In the case where nonminimal coupling makes the effective potential flatter than in the case of $p = 1$ and $\xi = 0$, assisted inflation occurs in the region of $\phi > 0$. For example, for $p = 2/3$ and $p = 1/2$ cases, the coupling ξ is required to be $\xi \gtrsim 3 \times 10^{-3}$ and $\xi \gtrsim 7 \times 10^{-3}$, respectively, to lead to assisted inflation while ϕ rolls down toward ϕ_*. Although it is rather difficult to obtain the sufficient number of e-foldings to solve the cosmological puzzles only by the effect of nonminimal coupling in the case of $p \leq 1$, it may be possible to lead to sufficient inflation in the multi-field case.

What we emphasize is that inflation is realized even in the one-field case with $p \leq 1$ in taking into account the effect of nonminimal coupling.

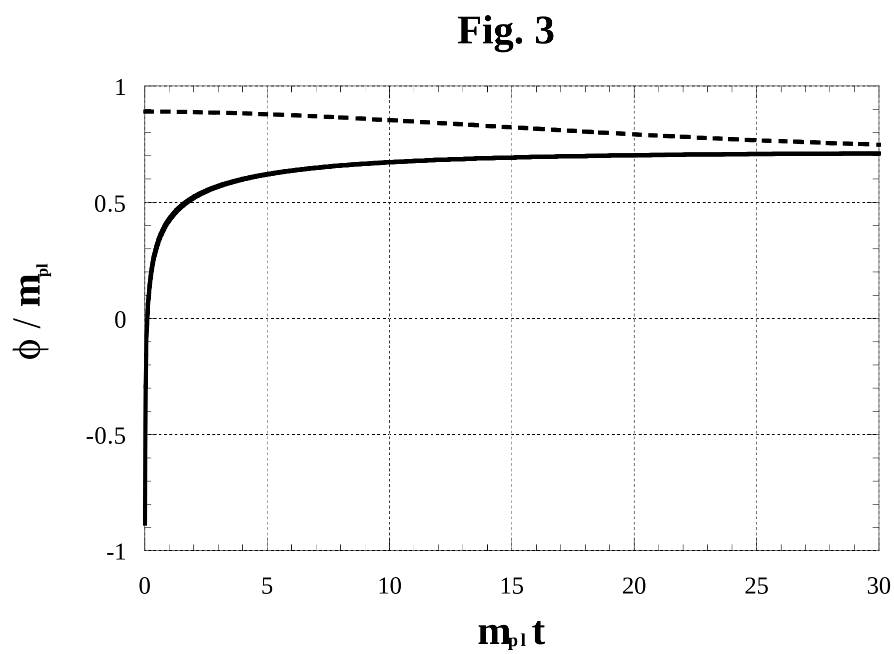


FIG. 3: The evolution of the inflaton field ϕ as a function of t in the case of $\xi = 0.05$ and $p = 1/2$ with the initial values of $\phi_i = -0.882 m_{\rm pl}$ (solid) and $\phi_i = 0.891 m_{\rm pl}$ (dotted). In both cases, inflaton evolves toward the potential minimum at $\phi_* = 0.714 m_{\rm pl}$.

B. Case of $\xi < 0$

When ξ is negative, the shape of the effective potential $\hat{V}(\phi)$ is different depending on the relation of ξ and p. In the case of $|\xi| < 1/2p$, the potential does not have any extrema. However, when $|\xi| > 1/2p$, it has both of a local minimum and a local maximum. In what follows, we examine these two different cases separately.

1. Case of $|\xi| < 1/2p$

In this case, $\hat{V}(\phi)$ decreases monotonically with the increase of ϕ. Although the potential is flatter than in the case of $\xi = 0$ for negative values of ϕ, it is steeper for $\phi > 0$ (See Fig. 1). Hence assisted inflation does not take place when ϕ is initially located in the region of $\phi > 0$. However, for the initial values of $\phi < 0$, we can expect inflation to occur even in the case of $p \leq 1$. For example, let us consider the case of $p = 3/4$ with the initial value of $\phi_i = -2m_{\rm pl}$. Numerical calculations show that we have an inflationary solution at the initial stage for the coupling of $|\xi| \gtrsim 0.3$. We depict in Fig. 4 the evolution of the scale factor a as a function of t for two cases of $\xi = -0.5$ and $\xi = -0.2$. Although the universe evolves as non-inflationary at the whole stage in the $\xi = -0.2$ case, inflation occurs in the $\xi = -0.5$ case at the initial stage where ϕ is negative. The larger values of $|\xi|$ ($\gtrsim 0.3$) lead to the larger amount of inflation. If ϕ is initially smaller than the value of $\phi_i = -2m_{\rm pl}$, one may consider that we will obtain the larger amount of e-foldings. However, this is not the case. With the decrease of ϕ, since the $-\xi\kappa^2\phi^2$ term in Eq. (2.19) increases and becomes

much larger than unity, the velocity of the ϕ field is larger than that of the Φ field. This suggests that the ϕ field does not evolve slowly enough to drive inflation for the smaller values of ϕ. For example, in the case of $p = 3/4$ and $\xi = -0.5$, we have numerically found that the universe evolves as non-inflationary for the values of $\phi_i \lesssim -4m_{\rm pl}$.

When p is less than unity, inflation is not realized unless we choose rather large values of $|\xi|$. For the case of $p = 1/2$ with the initial value of $\phi_i = -2m_{\rm pl}$, we require the values of $|\xi| \gtrsim 0.6$ for inflation to occur. However, we should stress that assisted inflation is possible in the case of $p \le 1$ for the larger values of $|\xi|$ which satisfy $|\xi| < 1/2p$.

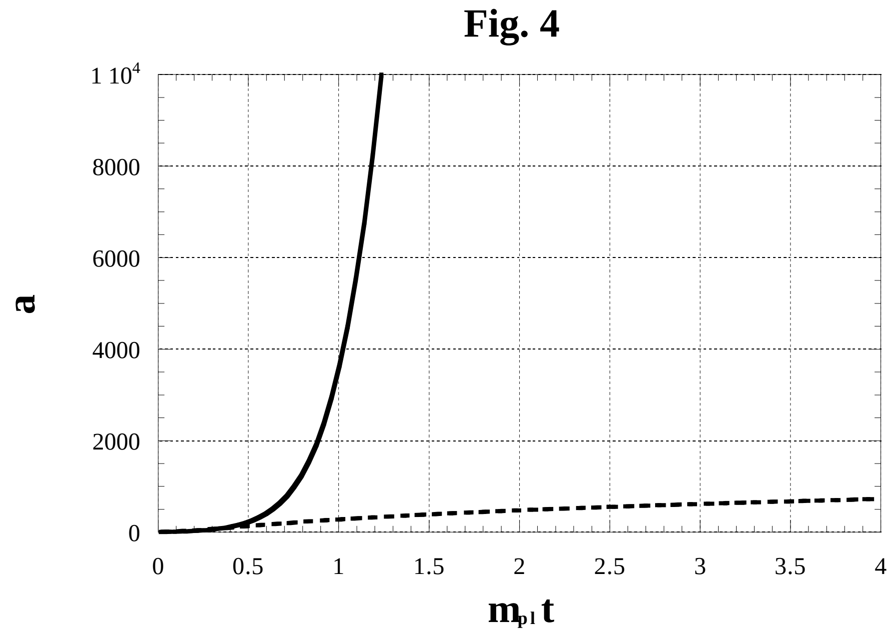


FIG. 4: The evolution of the scale factor a as a function of t in the case of $\xi = -0.5$ (solid) and $\xi = -0.2$ (dotted) with $p = 3/4$ and the initial value of $\phi_i = -2m_{\rm pl}$. In the case of $\xi = -0.5$, assisted inflation by nonminimal coupling relevantly occurs in the region of $\phi < 0$. On the other hand, when $\xi = -0.2$, inflation does not take place from the beginning.

2. Case of $|\xi| > 1/2p$

In this case, the potential $\hat{V}(\phi)$ is a local maximum at

$$\phi_1 = -\left(\sqrt{\frac{p}{4\pi}} - \sqrt{\frac{p}{4\pi} + \frac{1}{8\pi\xi}}\right) m_{\rm pl}, \tag{3.9}$$

and a local minimum at

$$\phi_2 = -\left(\sqrt{\frac{p}{4\pi}} + \sqrt{\frac{p}{4\pi} + \frac{1}{8\pi\xi}}\right) m_{\rm pl}. \tag{3.10}$$

Note that both of ϕ_1 and ϕ_2 are negative. With the existence of a local minimum, we can expect assisted inflation to occur in the region of $\phi < \phi_1$. We consider the case of $p = 1$ and $\xi = -1$ as one example. In this case, ϕ_1 and ϕ_2 correspond to $\phi_1 = -0.083m_{\rm pl}$ and $\phi_2 = -0.482m_{\rm pl}$, respectively. When ϕ initially lies in $\phi_1 < \phi_i < 0$, assisted inflation occurs when inflaton rolls down in the region of $\phi < 0$ as in the case of $|\xi| < 1/2p$. However, once inflaton passes through $\phi = 0$, assisted mechanism does not work since the effective potential becomes steeper compared with

the $\xi = 0$ case. When the initial value of inflaton is less than ϕ_1, the field evolves toward the local minimum at $\phi = \phi_2$ and is finally trapped in this minimum except the case where ϕ_i is much smaller than ϕ_2. Inflation relevantly occurs as ϕ approaches the potential minimum, since $\hat{V}(\phi)$ becomes gradually flatter. We show in Fig. 5 the evolution of the scale factor in the original frame for two cases of $\phi_i = -0.090 m_{\rm pl}$ and $\phi_i = -3 m_{\rm pl}$. In both cases, we find that the universe evolves as inflationary. Inflaton is finally trapped in the local minimum at $\phi = \phi_2$ after $\bar{t} \approx 4$ for the $\phi_i = -0.090 m_{\rm pl}$ case, and after $\bar{t} \approx 1.5$ for the $\phi_i = -3 m_{\rm pl}$ case. On the other hand, when the initial value of inflaton is $\phi_i \lesssim -5 m_{\rm pl}$, the ϕ field moves rather rapidly and goes beyond the local maximum at ϕ_1. In this case, since the velocity of the ϕ field is large due to the relation of Eq. (2.19), we have no inflationary solution. Namely, in the case of $p = 1$ and $\xi = -1$, assisted inflation is realized for the initial values of $-5 m_{\rm pl} \lesssim \phi_i < 0$.

If p is less than unity, we require rather large values of $|\xi|$ greater than the order of unity due to the condition of $|\xi| > 1/2p$. However, it is important to note that nonminimal coupling can lead to inflation even if $p \leq 1$ in the region of $\phi < 0$. We have numerically found that assisted inflation can be realized for the values of ξ which satisfy $|\xi| > 1/2p$.

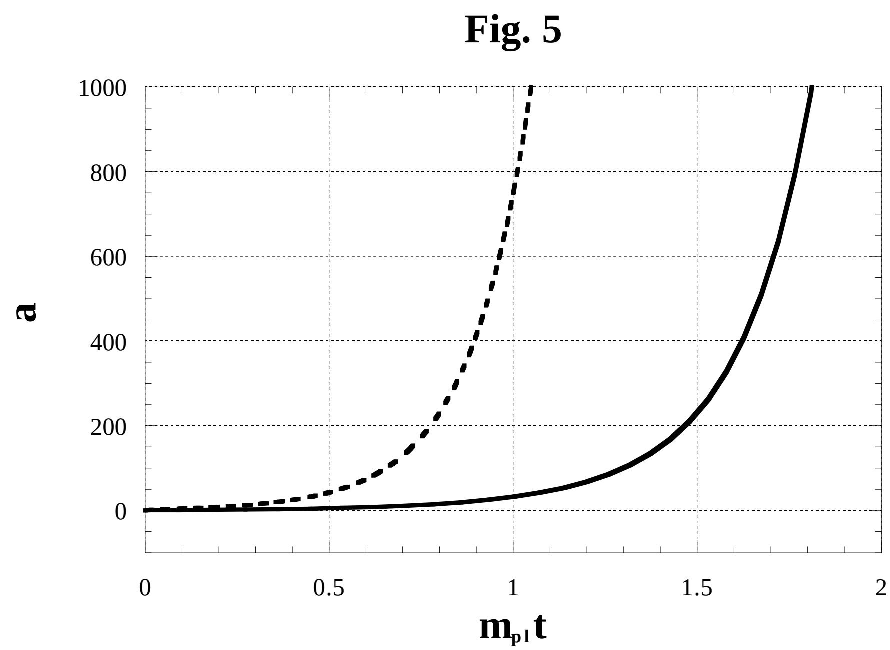
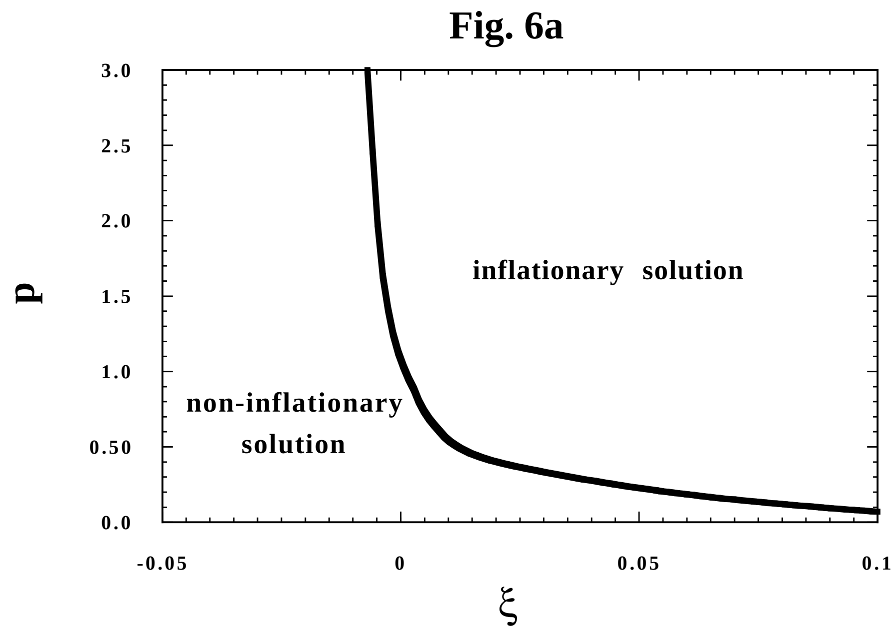


FIG. 5: The evolution of the scale factor a as a function of t in the case of $\xi = -1$ and $p = 1$ with the initial values of $\phi_i = -0.090 m_{\rm pl}$ (solid) and $\phi_i = -3 m_{\rm pl}$ (dotted). In both cases, inflation occurs while inflaton slowly evolves toward the potential minimum at $\phi_2 = -0.482 m_{\rm pl}$.

C. Summary

Finally, we present two-dimensional plots of ξ and p which divide the parameter regions of the inflationary and non-inflationary solutions in Fig. 6. These parameter spaces depend on initial values of inflaton. For $\phi_i \geq 0$, inflation can take place even when $p < 1$ for positive values of ξ (see Fig. 6a). In this case, however, negative ξ does not lead to assisted inflation because assisted mechanism is absent for $\phi > 0$. On the other hand, when inflaton is initially located for $\phi_i < 0$, inflationary behavior appears when inflaton evolves in the region of $\phi < 0$ even for negative ξ (see Fig. 6b).



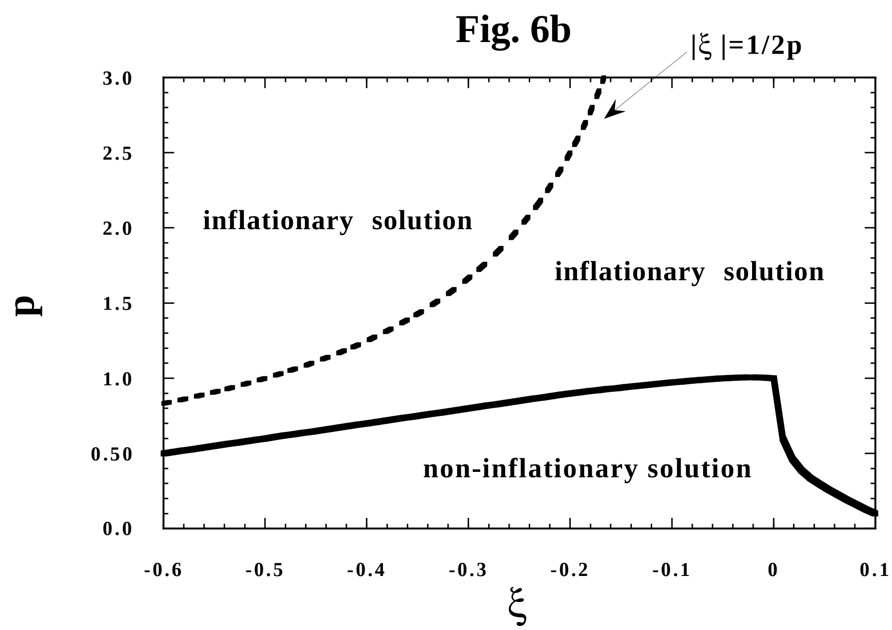


FIG. 6: The parameter regions of ξ and p where inflationary behavior appears for the initial values of $\phi_i = 0$ (Fig. 6a) and $\phi_i = -0.5m_{\rm pl}$ (Fig. 6b). In both figures, the solid curves separate the regions of inflationary and non-inflationary solutions. Assisted inflation can be realized for $\phi_i \geq 0$ in the case of $\xi > 0$. For $\phi_i < 0$, negative ξ also leads to assisted inflation.

IV. CONCLUDING REMARKS AND DISCUSSIONS

In this paper, we have investigated the dynamics of power-law inflation with a nonminimally coupled inflaton field. We studied how nonminimal coupling affects the dynamics of inflation with an exponential potential $V(\phi) =$

$V_0 \exp(-\sqrt{16\pi/pm_{\rm pl}^2}\phi)$ in the one-field model.

In the case of $\xi > 0$, since an effective potential (2.11) in the Einstein frame which appears by a conformal transformation becomes flatter for $\phi > 0$ than in the case of $\xi = 0$, assisted inflation can be realized by nonminimal coupling. In this case, nonminimal coupling gives rise to a potential minimum at some positive value of ϕ, and the potential becomes sufficiently flat around this area. Assisted mechanism works except the case where inflaton evolves in the region of $\phi < 0$ and ϕ is close to the value of $\phi_c = 1/\sqrt{8\pi\xi}$. Even when the power p is less than unity, we have an inflationary solution by choosing the appropriate values of ξ. For example, we have numerically found that inflation occurs for $\xi \gtrsim 3 \times 10^{-3}$ when $p = 2/3$, and for $\xi \gtrsim 7 \times 10^{-3}$ when $p = 1/2$.

When ξ is negative, the shape of the effective potential $\hat{V}(\phi)$ is different depending on two cases of $|\xi| < 1/2p$ and $|\xi| > 1/2p$. In the former case, $\hat{V}(\phi)$ is a monotonically decreasing function of ϕ. Since the potential is flat for $\phi < 0$ compared with the case of $\xi = 0$, assisted mechanism works in this region as long as ϕ is not too far from $\phi = 0$. However, nonminimal coupling prevents inflation for $\phi > 0$. In the latter case, the effective potential has a local maximum at ϕ_1 and a local minimum at ϕ_2 with $\phi_2 < \phi_1 < 0$. When ϕ is initially in the range of $\phi_1 < \phi < 0$, the assisted dynamics is the same as the former case. When ϕ is smaller than ϕ_1 initially, inflaton evolves toward the potential minimum at ϕ_2. In this case, nonminimal coupling assists inflation to occur unless ϕ is much smaller than ϕ_2. For summary, we conclude that assisted inflation can be realized for $\phi > 0$ in the case of $\xi > 0$; and for $\phi < 0$ in the case of $\xi < 0$.

The higher dimensional Kaluza-Klein theories often give rise to exponential potentials which are obtained by means of a conformal transformation to the Einstein frame. Then there is a possibility that inflaton is minimally coupled to gravity in the effective four-dimensional theories. Although we did not ask the origin of the exponential potential and considered a nonminimally coupled inflaton field in the four-dimensional action as in the chaotic inflation model plus nonminimal coupling, we have to keep in mind that nonminimal coupling may not play relevant roles in realistic models of physics. What we emphasize is that assisted inflation is possible even in the one-field model by introducing nonminimal coupling. When inflaton is minimally coupled to the spacetime curvature in the effective four-dimensional theories, inflation can be assisted by considering multiple scalar fields.

The problem of power-law inflation with an exponential potential is that the potential does not have a local minimum which causes a successful reheating. We find that introducing nonminimal coupling gives rise to a local minimum to which inflaton rolls down. However, since inflaton continues to possess a constant energy at this minimum, the universe inflates forever. This ever-inflating problem also occurs in the minimally coupled case. One exit mechanism is to introduce another scalar field as the hybrid inflation model, by which inflaton is to evolve toward a true minimum. Although we do not consider realistic models of exponential potentials which have graceful exit from inflation in this paper, it is of interest whether nonminimal coupling or multiple scalar fields assist inflation or not in such models. These issues are under consideration.

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